Fast Imaging of Energetic Neutral Atoms near Mars

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Overview

- Mars Express and Aspera-3
- Simulations of hydrogen and oxygen ENAs at Mars
- Fast generation of simulated ENA images
On ESA’s Mars Express mission
Launch June 2003 on Russian Soyuz/Fregat
Arrives in December 2003
ASPERA’s two ENA Instruments

**NPI** Neutral particle imager. Higher angular resolution. No energy resolution

**NPD** Neutral particle detector. Energy and mass resolution. New design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NPI</th>
<th>NPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particles to be measured</td>
<td>ENA</td>
<td>ENA</td>
</tr>
<tr>
<td>Energy range, keV per charge</td>
<td>≈0.1 - 60</td>
<td>0.1 - 10</td>
</tr>
<tr>
<td>Energy resolution, ΔE/E</td>
<td>No</td>
<td>0.8</td>
</tr>
<tr>
<td>Mass resolution,</td>
<td>No</td>
<td>H, O</td>
</tr>
<tr>
<td>Intrinsic field of view</td>
<td>9°×344°</td>
<td>9°×180°</td>
</tr>
<tr>
<td>Angular resolution (FWHM)</td>
<td>4.6°×11.5°</td>
<td>5°×30°</td>
</tr>
<tr>
<td>G-factor / pixel, cm²sr</td>
<td>2.5×10⁻³</td>
<td>6.2×10⁻³</td>
</tr>
<tr>
<td>Efficiency, ε, %</td>
<td>(ε not incl.)</td>
<td>(ε not incl.)</td>
</tr>
<tr>
<td>Time resolution (full 3D), s</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Mass, kg</td>
<td>0.7</td>
<td>1.3</td>
</tr>
<tr>
<td>Power, W</td>
<td>0.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Simulated ENA Images near Mars

- Parametric model of solar wind hydrogen ENA production
- How does the images depend on the parameters?
- Backscattering of ENAs by Mars’ atmosphere
- Planetary oxygen ENAs
- Image generation: First step toward image inversion
Proton Flow Streamlines

Kallio’s empirical model

Proton flux
+ Neutral density
⇒ ENA image
Neutral Densities

Densities of O, H\textsubscript{2} and H. Parameters: exobase density and temperature

\[ n_i(r) = N_i e^{-\beta_i \left( \frac{1}{r_0} - \frac{1}{r} \right)} \zeta(\beta_i / r) \]
ENA Images from $3 R_m$
ENA Images – Log Scale
Parameter Dependence

## Importance of Parameters

Average relative change in an image when a parameter change

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference value</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>H Exobase temperature</td>
<td>192 K</td>
<td>3.5</td>
</tr>
<tr>
<td>Solar wind velocity</td>
<td>400 km/s</td>
<td>1.6</td>
</tr>
<tr>
<td>Solar wind density</td>
<td>2.5 cm$^{-3}$</td>
<td>1.0</td>
</tr>
<tr>
<td>Position of the bow shock</td>
<td>1.55 $R_m$</td>
<td>1.0</td>
</tr>
<tr>
<td>H Exobase density</td>
<td>9.9 · $10^5$ cm$^{-3}$</td>
<td>0.92</td>
</tr>
<tr>
<td>Position of the magnetopause</td>
<td>1.2 $R_m$</td>
<td>0.67</td>
</tr>
<tr>
<td>Solar wind temperature</td>
<td>10 eV</td>
<td>0.63</td>
</tr>
<tr>
<td>H$_2$ Exobase temperature</td>
<td>192 K</td>
<td>0.083</td>
</tr>
<tr>
<td>Magnetopause penetration</td>
<td>1/6</td>
<td>0.064</td>
</tr>
<tr>
<td>H$_2$ Exobase density</td>
<td>3.8 · $10^6$ cm$^{-3}$</td>
<td>0.013</td>
</tr>
<tr>
<td>O (hot) Exobase temperature</td>
<td>4.4 · $10^3$ K</td>
<td>0.011</td>
</tr>
<tr>
<td>O (hot) Exobase density</td>
<td>5.5 · $10^3$ cm$^{-3}$</td>
<td>0.0042</td>
</tr>
<tr>
<td>O (thermal) Exobase temperature</td>
<td>173 K</td>
<td>0.00078</td>
</tr>
<tr>
<td>O (thermal) Exobase density</td>
<td>1.4 · $10^8$ cm$^{-3}$</td>
<td>0.00019</td>
</tr>
</tbody>
</table>
ENA Albedo

- ENAs from charge-exchange will precipitate on Mars
- Some backscattered after interaction with the atmosphere
- Complicated backscattering function
- Simplification: 58% backscattered, isotropic
Effects of ENA Albedo
Imaging of Planetary Oxygen

- Oxygen ions from photoionization, accelerated by $E$ and $B$ fields
- Kinetic equation, with sources and sinks, solved for the $O^+$ distribution
- Asymmetric distribution
- Charge-exchange with H, H$_2$ and O
- Oxygen ENAs mostly below 600 eV
  Flux $10^5$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ eV$^{-1}$ in the energy range 0.1-1.7 keV
- Imaging feasible. Gives estimate of oxygen escape
Some Comments on the Charge-Exchange Approximation

H$^+$ (velocity $v$) - H (at rest) interaction.

Two idealized outcomes:
1. $\theta = 0$: H (at rest), H$^+$ (velocity $v$)
2. $\theta = 180$: H$^+$ (at rest), H (velocity $v$)

In reality we have the whole continuum of possible deflections ($0 \leq \theta \leq 180$)
Differential cross sections by solving the Schrödinger equation
Differential Cross Section

Collision peak at $\theta = 0$. CXE at $\theta = 180$

Idealized charge-exchange valid over 50 eV in our simulations
Fast Generation of Simulated ENA Images

Reasons for fast image generation:

1. Parameter extraction from genuinely non-linear emission models require the generation of many images (for different parameters)

2. A general problem in computer graphics: Volume rendering
Requirements on the Algorithm

We assume only volume emissions (no surfaces), without attenuation

Requirements

1. Speed. Minimize the number of function (radiosity) evaluations

2. Error control. The ability to trade error for time

3. Memory. Minimal memory requirements
The Generation of Simulated ENA Images

Volume emissions (radiosity) of ENAs emitted from \( r \) in direction \( d \) is 
\( g(r, d) \) \([1/(m^3 \text{ sr s})]\).

The radiance (or intensity) is

\[
f(r, d) = \int_0^\infty g(r + sd, -d) \, ds \quad [1/(m^2 \text{ sr s})]
\]

From now on we assume that the observer is at the origin
\( \Rightarrow g(r) \) and \( f(d) \)
Irradiance

The received flux (irradiance) from a finite solid angle $\Omega$, e.g., corresponding to an image pixel, can be formulated in two ways

1. As a two-dimensional integral over the solid angle,

$$ F(\Omega) = \int_{\Omega} f(\mathbf{d}) \, d\Omega \quad [1/(\text{m}^2 \text{ s})], \text{ or} $$

2. As a three-dimensional integral over the volume corresponding to $\Omega$,

$$ F(\Omega) = \int_{V} \frac{g(\mathbf{r})}{r^2} \, dV \quad [1/(\text{m}^2 \text{ s})]. $$
Irradiance Geometry

\[ g(r) \]

\[ f(d) \]

\[ \Omega \]

\[ V \]
**Pixel Intensity**

For a pixel, $i$, with a corresponding solid angle, $\Omega_i$, we thus have two approaches to compute the average radiance,

$$\bar{f}_i = F(\Omega_i)/\Omega_i, \quad [1/(m^2 \text{ sr s})]$$

Denote them LOS (solid angle/surface integration) and VOL (volume integration).

In practice, the integrals are evaluated by numerical quadrature. Using the midpoint quadrature rule for LOS, $\bar{f}_i \approx f(d_{CM})$, gives the straightforward, classical, way of generating an image of a radiation field, where we for each pixel do a line of sight (LOS) integration in the direction corresponding to a pixel.
Perspective, polar and canonical view volumes. Grids on image planes.

Canonical image:

\[ f(x, y) = \int_0^1 g(x, y, z) \, dz \]

1D analogue:

\[ f(x) = \int_0^1 g(x, z) \, dz \]
1D Model Problem

The pixels are intervals on the $x$-axis, and the pixel $i$’s radiosity is

$$\bar{f}_i = \int_{x_{i-1}}^{x_i} f(x) \, dx = \int_{x_{i-1}}^{x_i} \int_0^1 g(x, z) \, dz \, dx,$$

an integration over the rectangle $[x_{i-1}, x_i] \times [0, 1]$ in the $xz$-plane.
Different Algorithms

1. Fix step line of sight integration. One per pixel

2. Adaptive step line of sight integration. Adaptive number of lines per pixel. Correlations between the lines exist

3. Hierarchical, wavelet based, representation. Total work proportional to the number of cells, $N_s$. Error of the representation controlled by a threshold, $\epsilon$
Construction from coarse to fine grid. Cell values are predicted from coarser grid by polynomial subdivision. If the difference between a value by quadrature and by subdivision is smaller than $\epsilon$ we stop refining that sub-tree. The result is a quad tree of cell averages. Note that any cell average can be computed by this conservative subdivision.
Projection

Project the 2D representation (quad tree) onto a 1D representation (binary tree). Subdivide/sum to get pixel values.

Free parameters:

- Quadrature rule: function evaluations/accuracy
- Conservative subdivision: Order. Method
Advantages of the Hierarchical Method

- Total work and memory requirement is proportional to $N_s$
- Accuracy is specified by $\epsilon$
- Progressive rendering possible
Fast Volume Rendering Example

Reference 64×64 image and hierarchical image using 4032 function evaluations (less than one per pixel). The error is 6%.
An order of magnitude faster than fix step LOS
1% error with 20 000 function evaluations (5 per pixel)
Error Control

Accuracy can be traded for speed by adjusting $\epsilon$
Order of Accuracy

The error is proportional to

\[ e_{\text{VOL}} \sim \frac{1}{N_s} \quad \text{and} \quad \]

\[ e_{\text{LOS}} \sim \frac{1}{N^{2/3}} = \frac{1}{n^2} = h^2, \]

where \( N_s \) is the number of function evaluations for VOL, and \( N \) for LOS. The number of cells in each dimension for LOS is \( n \), of length \( h \).
Ongoing and Future Work

• Inversion techniques for parameter extraction from Aspera-3 data in 2004

• Optimized fast volume rendering

• Simulated very low energy neutral atom (VLENA) images of sputtered ENAs at Mercury

• Algorithms for phase space density computations for neutrals at Mercury

• Papers at http://vega.irf.se